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Bell's local causality, Leggett's crypto-nonlocality, and quantum separability are genuinely different concepts

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I clarify here the relation between Leggett's concept of crypto-nonlocality and the better known notions of Bell's local causality and quantum separability, emphasizing that these are three genuinely different concepts. In particular, I show that while the correlations of separable quantum states clearly satisfy the assumptions of crypto-nonlocality, the opposite is not true: there exist entangled states whose correlations are always compatible with Leggett's crypto-nonlocality.

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I. LEGGETT'S CRYPTO-NONLOCALITY

The concept of "crypto-nonlocality" [1] was introduced by Leggett in an attempt to explain quantum correlations with some kind of "realistic" picture: roughly speaking, it says that all individual subsystems of a composite system should locally behave as if they were in a pure quantum state, with well-defined properties.

To make it more precise, consider (following Leggett) the simplest case of bipartite correlations obtained from projective measurements on a two-qubit state. The measurement settings can be described by unit vectors $\vec{a}$ and $\vec{b}$ on the Bloch sphere $\mathcal{S}^2$, and the measurement outcomes are binary variables, denoted here by $\alpha,\beta = \pm 1$. The correlation observed by the two parties, Alice and Bob, is then described by the joint conditional probability distribution $P(\alpha,\beta|\vec{a},\vec{b})$. According to Leggett's assumptions, it should be possible to decompose this correlation as a mixture of correlations $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ whose local marginal probability distributions for Alice and Bob are those corresponding to pure qubits in the states $|\vec{u}\rangle$ and $|\vec{v}\rangle$, respectively, represented by unit vectors $\vec{u}$ and $\vec{v}$ on the Bloch sphere. That is, the correlation $P(\alpha,\beta|\vec{a},\vec{b})$ is compatible with Leggett's crypto-nonlocality *if and only if* there exists a non-negative, normalized probability distribution $\rho(\vec{u},\vec{v})$ and correlations $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ ($\geqslant 0$) such that

$$P(\alpha,\beta|\vec{a},\vec{b}) = \int_{\mathcal{S}^2}\int_{\mathcal{S}^2} P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})\,\rho(\vec{u},\vec{v})\,d\vec{u}\,d\vec{v}, \tag{1}$$

with the marginals of $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ satisfying

$$M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b}) := \sum_{\alpha,\beta} \alpha\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b}) \;=\; \vec{u}\cdot\vec{a}, \tag{2}$$

$$M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b}) := \sum_{\alpha,\beta} \beta\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b}) \;=\; \vec{v}\cdot\vec{b} \tag{3}$$

for all $\vec{u},\vec{v}$, and for all measurement settings $\vec{a},\vec{b}$ under consideration. In the case of polarization, as initially considered by Leggett [1], Eqs. (2) and (3) impose that the local observations, conditioned on the "hidden variables" $\vec{u},\vec{v}$, should satisfy Malus' law.

Equations (1)–(3) are the only constraints imposed by the assumptions of crypto-nonlocality [2], as defined by Leggett in Ref. [1]. Note that the correlations $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ in the decomposition (1) are nonsignaling, and that no time-ordering of Alice and Bob's measurements needs to be specified [3]. Only Alice and Bob's local marginals $M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b})$ and $M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b})$ are constrained by the assumptions of crypto-nonlocality, through Eqs. (2) and (3). Nothing is said about the correlation terms $C_{\vec{u},\vec{v}}(\vec{a},\vec{b}) := \sum_{\alpha,\beta}\alpha\beta\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$, which can in particular still make the correlations $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$—and hence $P(\alpha,\beta|\vec{a},\vec{b})$—violate Bell's assumption of local causality [4] (see below).

It is worth emphasizing also that the crypto-nonlocality constraints (1)–(3) are defined for specific measurement settings $\vec{a},\vec{b} \in \mathcal{S}^2$. This is in stark contrast with Bell's local causality assumption, for instance, where the measurement settings are just arbitrary labels; this is, however, analogous to the case in which one asks whether a given two-qubit correlation is compatible with a separable state, as the measurement settings must in general also be specified. Note further that the correlations obtained from a quantum state can be compatible with the crypto-nonlocality constraints for certain measurement settings, but may cease to satisfy them if more settings are considered.

As Leggett indeed showed, quantum theory predicts certain correlations which do not satisfy the constraints (1)–(3); the canonical example is the correlation obtained from the singlet state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, for which $P(\alpha,\beta|\vec{a},\vec{b}) = \frac{1}{4}(1-\alpha\beta\,\vec{a}\cdot\vec{b})$. One can in fact derive, from the constraints (1)–(3) and the non-negativity of probability distributions only, and for some specific measurement settings, so-called *Leggett inequalities* which can be violated by quantum theory and can be tested experimentally [1,3,5–8]. All experiments to date [3,5–7,9,10] have shown (up to a few loopholes) a violation of Leggett's crypto-nonlocality, and have been in agreement with quantum predictions.

II. LEGGETT'S CRYPTO-NONLOCALITY VERSUS BELL'S LOCAL CAUSALITY

It is quite natural to compare the constraints imposed by the assumptions of crypto-nonlocality to those of Bell's local causality assumption [4]—a very natural assumption to explain correlations between distant events, but famously incompatible with quantum correlations.

As it turns out, there is in fact no logical relation between the two notions; correlations can independently be compatible or incompatible with Leggett's constraints, and compatible or incompatible with Bell's assumption. Let me clarify this with the following explicit examples:

(i) The fully random correlation $P(\alpha,\beta|\vec{a},\vec{b}) = \frac{1}{4}$ for all $\alpha,\beta,\vec{a},\vec{b}$ is compatible both with Leggett's crypto-nonlocality [take, e.g., $\vec{u}$ and $\vec{v}$ independently and uniformly distributed on $\mathcal{S}^2$, and define $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b}) = \frac{1}{4}(1+\alpha\,\vec{u}\cdot\vec{a})(1+\beta\,\vec{v}\cdot\vec{b})$] and with Bell's local causality.

(ii) When all (or sufficiently many and well-chosen) measurement settings $\vec{a},\vec{b} \in \mathcal{S}^2$ are considered, the singlet state correlations $P(\alpha,\beta|\vec{a},\vec{b}) = \frac{1}{4}(1-\alpha\beta\,\vec{a}\cdot\vec{b})$ are incompatible with both Leggett's crypto-nonlocality [1] and Bell's local causality [4].

(iii) However, when the measurements settings under consideration are restricted, for instance to the equatorial plane of the Bloch sphere, the singlet state correlations are compatible with Leggett's crypto-nonlocality [1], but they can violate Bell's local causality [4,11]. Another, nonquantum correlation that is compatible with Leggett's assumption but violates Bell's local causality is the "PR-box" correlation [12] $P(\alpha,\beta|\vec{a}_i,\vec{b}_j) = \frac{1}{4}[1+\alpha\beta\,(-1)^{ij}]$, for two measurement settings $(i,j = 0,1)$ for both Alice and Bob (take $\vec{u}$ to be orthogonal to both $\vec{a}_0$ and $\vec{a}_1$, and $\vec{v}$ to be orthogonal to both $\vec{b}_0$ and $\vec{b}_1$).

(iv) Lastly, consider deterministic correlations $P(\alpha,\beta|\vec{a}_i,\vec{b}) = \delta_{\alpha,1}\,\delta_{\beta,1}$ (where $\delta_{i,j}$ is the Kronecker delta) for at least two different settings $\vec{a}_0$ and $\vec{a}_1$ for Alice: such correlations are incompatible with Leggett's crypto-nonlocality [all $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ in the decomposition (1) must indeed be such that $M^A_{\vec{u},\vec{v}}(\vec{a}_0,\vec{b}) = M^A_{\vec{u},\vec{v}}(\vec{a}_1,\vec{b}) = \vec{u}\cdot\vec{a}_0 = \vec{u}\cdot\vec{a}_1 = 1$, which is impossible for $\vec{a}_0 \neq \vec{a}_1$], while they clearly satisfy Bell's local causality assumption. Note also with this example that—in contrast to Bell's local causality—Leggett's crypto-nonlocality can in principle be falsified by considering only one party [13].

However artificial these last examples may look (e.g., PR-box correlations are not usually thought of as having Bloch vectors as "measurement settings"), they illustrate indeed the independence of the two notions of crypto-nonlocality and local causality. Note that the same observations hold when comparing Leggett's crypto-nonlocality to the concept of quantum steering, a weaker notion of quantum nonlocality [14] (for the last example, consider, e.g., just one setting for Bob, and nonsteerability from Alice to Bob).

III. LEGGETT'S CRYPTO-NONLOCALITY VERSUS QUANTUM SEPARABILITY

The next natural question to ask is how Leggett's crypto-nonlocality compares to the notions of quantum separability and quantum entanglement. First of all, note that correlations from two-qubit separable states obviously satisfy Leggett's assumptions (1)–(3) of crypto-nonlocality. Reciprocally, one can already see from the previous remarks that some quantum correlations can be compatible with Leggett's assumptions, but they violate Bell's local causality (cf. the third example above). These correlations can therefore not be generated by separable quantum states, which shows that crypto-nonlocality and quantum separability are not equivalent concepts.

Note that the argument so far considers only a limited number of measurement settings on the two-qubit singlet state; indeed, as noted before, when all measurements are allowed, the statistics of the singlet state are incompatible with Leggett's assumption of crypto-nonlocality. Could it then be that when all measurement settings are allowed, the correlations from any entangled state are incompatible with Leggett's constraints?

The answer is no. I show in the Appendix how to construct an explicit "Leggett model" that reproduces the correlations $P(\alpha,\beta|\vec{a},\vec{b}) = \frac{1}{4}(1-\alpha\beta\,V\,\vec{a}\cdot\vec{b})$ of two-qubit Werner states [15] $\varrho_V = V\,|\Psi^-\rangle\langle\Psi^-| + (1-V)\,\frac{\mathbb{1}}{4}$ for all $V \in [0,\frac{1+1/\sqrt{2}}{2}]$, and for all settings $\vec{a},\vec{b} \in \mathcal{S}^2$. Now, Werner states are entangled for $V > 1/3$: for $1/3 < V \leqslant \frac{1+1/\sqrt{2}}{2} \simeq 0.85$, they thus provide an example of entangled states that are compatible with Leggett's crypto-nonlocality for all projective measurements [16,17]. This reinforces the claim that crypto-nonlocality and quantum separability are genuinely different notions: quantum separability implies crypto-nonlocality, but not reciprocally.

IV. DISCUSSION

The objective of this paper was to clarify the fact that Leggett's notion of crypto-nonlocality is quite different from those of local causality and quantum separability. Note that this conclusion may be challenged if one does not only stick to Leggett's definition of crypto-nonlocality through Eqs. (1)–(3), but invokes rather some particular (physically motivated) interpretation and introduces additional assumptions [18,19]: it has indeed been argued [18] that the physical intuition that motivates Leggett's model may actually lead to quantum separability rather than to Leggett's definition [Eqs. (1)–(3) only]. However, it is in fact precisely because Leggett freed himself from certain "physical intuitions" that the notion of crypto-nonlocality that he introduced is not—as emphasized here—simply equivalent to quantum separability. One may find Leggett's notion itself unnatural (when not augmented by additional assumptions), may question whether an explanation of correlations based on crypto-nonlocality would then be satisfying, and may thereby dispute its physical relevance [18]. Nevertheless, the study of Leggett's assumptions has already inspired some interesting results, e.g., on the predictive power and the completeness of quantum theory [3,20], and it is likely to generate more insights in the future about the foundations of quantum theory.

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APPENDIX

1. Bipartite, binary-outcome correlations

A convenient way to write any bipartite correlation $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ [as in the decomposition (1)] with binary outcomes $\alpha,\beta = \pm 1$ is

$$P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b}) = \tfrac{1}{4}\big[1 + \alpha\,M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b}) + \beta\,M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b}) + \alpha\beta\,C_{\vec{u},\vec{v}}(\vec{a},\vec{b})\big], \tag{A1}$$

where $M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b}) = \sum_{\alpha,\beta} \alpha\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ and $M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b}) = \sum_{\alpha,\beta} \beta\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ are the marginals on Alice and Bob's side, while $C_{\vec{u},\vec{v}}(\vec{a},\vec{b}) = \sum_{\alpha,\beta} \alpha\beta\, P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ is the correlation coefficient. The constraint that the probabilities $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ must be non-negative, for all α and β, is equivalent to

$$-1 + \left|M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b}) + M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b})\right| \leqslant C_{\vec{u},\vec{v}}(\vec{a},\vec{b}) \leqslant 1 - \left|M^A_{\vec{u},\vec{v}}(\vec{a},\vec{b}) - M^B_{\vec{u},\vec{v}}(\vec{a},\vec{b})\right|. \tag{A2}$$

2. An explicit Leggett model for two-qubit Werner states of visibility $V \leqslant \frac{1+1/\sqrt{2}}{2}$

Let us choose $\vec{v} = -\vec{u}$, with $\vec{u}$ uniformly distributed on the Bloch sphere $\mathcal{S}^2$, and assume that the correlations $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ satisfy the crypto-nonlocality constraints (2) and (3). After integrating over $\vec{u}$, we find, for all $\vec{a},\vec{b}$,

$$M^A(\vec{a},\vec{b}) := \int_{\mathcal{S}^2} M^A_{\vec{u},-\vec{u}}(\vec{a},\vec{b})\,\frac{d\vec{u}}{4\pi} = \int_{\mathcal{S}^2} (\vec{u}\cdot\vec{a})\,\frac{d\vec{u}}{4\pi} = 0,$$
$$M^B(\vec{a},\vec{b}) := \int_{\mathcal{S}^2} M^B_{\vec{u},-\vec{u}}(\vec{a},\vec{b})\,\frac{d\vec{u}}{4\pi} = \int_{\mathcal{S}^2} (-\vec{u}\cdot\vec{b})\,\frac{d\vec{u}}{4\pi} = 0,$$

which are the marginals expected for the correlations of the Werner state $\varrho_V = V\,|\Psi^-\rangle\langle\Psi^-| + (1-V)\,\frac{\mathbb{1}}{4}$. To obtain the full correlations of the Werner state, we also need the correlation coefficient to be

$$C(\vec{a},\vec{b}) := \int_{\mathcal{S}^2} C_{\vec{u},-\vec{u}}(\vec{a},\vec{b})\,\frac{d\vec{u}}{4\pi} = -V\,\vec{a}\cdot\vec{b}. \tag{A3}$$

Now, Eq. (A2) reads

$$-1 + |\vec{u}\cdot(\vec{a}-\vec{b})| \leqslant C_{\vec{u},-\vec{u}}(\vec{a},\vec{b}) \leqslant 1 - |\vec{u}\cdot(\vec{a}+\vec{b})|. \tag{A4}$$

After integrating it over the values of $\vec{u}$ [with $\int_{\mathcal{S}^2} |\vec{u}\cdot(\vec{a}\pm\vec{b})|\,\frac{d\vec{u}}{4\pi} = \frac{1}{2}||\vec{a}\pm\vec{b}|| = \sqrt{\frac{1\pm\vec{a}\cdot\vec{b}}{2}}$] and using the constraint (A3), this gives the necessary condition that

$$-1 + \sqrt{\frac{1-\vec{a}\cdot\vec{b}}{2}} \leqslant -V\,\vec{a}\cdot\vec{b} \leqslant 1 - \sqrt{\frac{1+\vec{a}\cdot\vec{b}}{2}}, \tag{A5}$$

which indeed holds for all values of $\vec{a}\cdot\vec{b} \in [-1,1]$ (i.e., all unit vectors $\vec{a},\vec{b} \in \mathcal{S}^2$) when $0 \leqslant V \leqslant \frac{1+1/\sqrt{2}}{2}$.

For such values of V, it is always possible to find functions $C_{\vec{u},-\vec{u}}(\vec{a},\vec{b})$ satisfying (A3) and (A4) for all $\vec{a}$, $\vec{b}$ and $\vec{u}$; one can choose, for instance,

$$C_{\vec{u},-\vec{u}}(\vec{a},\vec{b}) = p^-_{\vec{a},\vec{b}}\,[-1 + |\vec{u}\cdot(\vec{a}-\vec{b})|] + p^+_{\vec{a},\vec{b}}\,[1 - |\vec{u}\cdot(\vec{a}+\vec{b})|]$$
$$\text{with } p^\pm_{\vec{a},\vec{b}} = \frac{1 - \sqrt{\frac{1\mp\vec{a}\cdot\vec{b}}{2}} \mp V\,\vec{a}\cdot\vec{b}}{2 - \sqrt{\frac{1+\vec{a}\cdot\vec{b}}{2}} - \sqrt{\frac{1-\vec{a}\cdot\vec{b}}{2}}}, \tag{A6}$$

such that for all $\vec{a},\vec{b}$, $p^+_{\vec{a},\vec{b}} + p^-_{\vec{a},\vec{b}} = 1$ and (for $V \leqslant \frac{1+1/\sqrt{2}}{2}$) $p^\pm_{\vec{a},\vec{b}} \geqslant 0$. This, together with the constraints (2) and (3) on the marginals, then defines valid (i.e., non-negative) probabilities $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ through Eq. (A1), and thus provides an explicit Leggett model for all projective measurements on two-qubit Werner states ϱ_V of visibility $V \leqslant \frac{1+1/\sqrt{2}}{2}$.

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[2] In his paper [1], Leggett also introduced another level of "hidden variables" λ's, and further decomposed $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ in the form $\int P_{\vec{u},\vec{v};\lambda}(\alpha,\beta|\vec{a},\vec{b})\rho_{\vec{u},\vec{v}}(\lambda)d\lambda$. As no further constraint was imposed on this "lower-level" decomposition, one actually does not need to consider it to define the notion of crypto-nonlocality. Note that introducing such variables λ's can make a model of the form (1) deterministic, as it is the case e.g. for the explicit Leggett models introduced in Ref. [1] and in the Supplementary Information of Ref. [5]. Note also that for a given λ, the correlation $P_{\vec{u},\vec{v};\lambda}(\alpha,\beta|\vec{a},\vec{b})$ can in general be signaling, while after averaging over the λ's, $P_{\vec{u},\vec{v}}(\alpha,\beta|\vec{a},\vec{b})$ remains non-signaling.

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[13] Note, however, that the deterministic correlations $P(\alpha,\beta|\vec{a}_i,\vec{b}) = \delta_{\alpha,1}\delta_{\beta,1}$ considered in the example cannot be obtained quantum mechanically from qubit measurements with $\vec{a}_0 \neq \vec{a}_1$; hence, it cannot be used to falsify Leggett's crypto-nonlocality assumption in a (quantum) experiment where the implementation of the measurements is trusted.

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Besides, as shown in the supplementary information of Ref. [3], Werner states ϱ_V with $V > \frac{\sqrt{3}}{2} \simeq 0.866$ violate a Leggett inequality. It remains an open question whether for $\frac{1+1/\sqrt{2}}{2} < V \leqslant \frac{\sqrt{3}}{2}$ Werner state correlations are compatible with Leggett's crypto-nonlocality.

[17] One may also ask whether the correlations obtained from positive operator-valued measurements (POVMs) on Werner states satisfy a generalized version of crypto-nonlocality, which would also consider general POVMs—the constraints (1)–(3) can indeed straightforwardly be generalized to POVMs, although I am not aware of any work in this direction. I conjecture that there is indeed a range of entangled Werner states that would still be compatible with that generalized version of crypto-nonlocality (analogously, POVMs on Werner states admit a locally causal model for $V \leqslant \frac{5}{12}$ [25]).

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